Linear Operations Using Masks

Masks are patterns used to define the weights used in averaging the neighbors of a pixel to compute some result at that pixel.

Expressing linear operations on neighborhoods

- many operations defined by cross correlation
- smoothing of image
- edge detection
  - ad hoc first derivative masks
  - ad hoc second derivative masks
  - LOG and DOG masks
- directional derivative masks
- special basis vector expansion of neighborhoods
  - look for energy matching certain patterns
  - Frei-Chen basis (const, edge, ripple, line, Laplacian)
  - Fourier (sine waves), Gabor wavelets
  - Laws texture masks (Ch 7)
Smoothing an image by averaging neighbors (boxcar)

Output pixel is the dot product of the input neighborhood and the mask
Properties of smoothing masks

- Coordinates of smoothing masks are positive and sum to one so that output on constant regions is the same as the input.
- The amount of smoothing and noise reduction is proportional to the mask size.
- Step edges are blurred in proportion to the mask size.

\[
\begin{align*}
\text{boxcar} & : \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} & \text{Gaussian} & : \frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \\
\end{align*}
\]

Gaussian has better properties than the boxcar.

Types of ideal edges (in 1D)

- upward step edge
- downward step edge
- upward ramp
- impulse or line
- teat

These types are also present in 2D and 3D images and are complicated by orientation variations.
**Boxcar smoothing filter example**

- **Box smoothing mask** $M = [1/3, 1/3, 1/3]$

<table>
<thead>
<tr>
<th>$S_1$</th>
<th>12</th>
<th>12</th>
<th>12</th>
<th>12</th>
<th>12</th>
<th>24</th>
<th>24</th>
<th>24</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1 \otimes M$</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>16</td>
<td>20</td>
<td>24</td>
<td>24</td>
<td>24</td>
</tr>
</tbody>
</table>

(a) $S_1$ is an upward step edge

<table>
<thead>
<tr>
<th>$S_4$</th>
<th>12</th>
<th>12</th>
<th>12</th>
<th>12</th>
<th>24</th>
<th>12</th>
<th>12</th>
<th>12</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_4 \otimes M$</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>

(d) $S_4$ is a bright impulse or “line”

So, reducing noise will also degrade the signal.

**Gaussian smoothing**

- **Gaussian smoothing mask** $M = [1/4, 1/2, 1/4]$

<table>
<thead>
<tr>
<th>$S_1$</th>
<th>12</th>
<th>12</th>
<th>12</th>
<th>12</th>
<th>12</th>
<th>24</th>
<th>24</th>
<th>24</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1 \otimes M$</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>15</td>
<td>21</td>
<td>24</td>
<td>24</td>
<td>24</td>
</tr>
</tbody>
</table>

(a) $S_1$ is an upward step edge

<table>
<thead>
<tr>
<th>$S_4$</th>
<th>12</th>
<th>12</th>
<th>12</th>
<th>12</th>
<th>24</th>
<th>12</th>
<th>12</th>
<th>12</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_4 \otimes M$</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>15</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>

(d) $S_4$ is a bright impulse or “line”
Differencing used to estimate 1\textsuperscript{st} and 2\textsuperscript{nd} derivatives

First differences

Second differences

Masks represent the first and 2\textsuperscript{nd} differences

Step edges X mask [-1, 0, +1]

(a) $S_1$ is an upward step edge

(b) $S_2$ is a downward step edge

Step edge is detected well, but edge location imprecise.
Ramp and impulse X mask \([-1, 0, +1]\)

\[
\begin{array}{c|cccccccccc}
S_3 & 12 & 12 & 12 & 12 & 15 & 18 & 21 & 24 & 24 & 24 \\
S_3 \otimes M & 0 & 0 & 0 & 3 & 6 & 6 & 6 & 3 & 0 & 0 \\
\end{array}
\]

(c) \(S_3\) is an upward ramp

\[
\begin{array}{c|cccccccccc}
S_4 & 12 & 12 & 12 & 12 & 24 & 12 & 12 & 12 & 12 & 12 \\
S_4 \otimes M & 0 & 0 & 0 & 12 & 0 & -12 & 0 & 0 & 0 & 0 \\
\end{array}
\]

(d) \(S_4\) is a bright impulse or “line”

Ramp edge now yields a broad weak response. Impulse response is a “whip”, first up and then down.

---

2nd derivative using mask \([-1, 2, -1]\)

\[
\begin{array}{c|cccccccccc}
S_1 & 12 & 12 & 12 & 12 & 12 & 24 & 24 & 24 & 24 & 24 \\
S_1 \otimes M & 0 & 0 & 0 & 0 & -12 & 12 & 0 & 0 & 0 & 0 \\
\end{array}
\]

(a) \(S_1\) is an upward step edge

\[
\begin{array}{c|cccccccccc}
S_2 & 24 & 24 & 24 & 24 & 24 & 12 & 12 & 12 & 12 & 12 \\
S_2 \otimes M & 0 & 0 & 0 & 0 & 12 & -12 & 0 & 0 & 0 & 0 \\
\end{array}
\]

(b) \(S_2\) is a downward step edge

Response is zero on constant region and a “double whip” amplifies and locates the step edge.
2nd derivative using mask [-1, 2, -1]

\[
\begin{array}{c|cccccccc}
S_3 & 12 & 12 & 12 & 12 & 15 & 18 & 21 & 24 \\
\hline
M & 0 & 0 & -3 & 0 & 0 & 0 & 3 & 0
\end{array}
\]

(c) \( S_3 \) is an upward ramp

\[
\begin{array}{c|cccccccc}
S_4 & 12 & 12 & 12 & 24 & 12 & 12 & 12 & 12 \\
\hline
M & 0 & 0 & 0 & -12 & 24 & -12 & 0 & 0
\end{array}
\]

(d) \( S_4 \) is a bright impulse or “line”

Weak response brackets the ramp edge. Bright impulse yields a double whip with gain of 3X original contrast.

---

Estimating 2D image gradient

Can estimate the column (x) gradient across 3 columns

\[
\frac{\partial f}{\partial x} \equiv f_x \approx \frac{1}{3} \left[ \frac{I[x+1,y] - I[x-1,y]}{2} + \frac{I[x+1,y-1] - I[x-1,y-1]}{2} + \frac{I[x+1,y+1] - I[x-1,y+1]}{2} \right]
\]

Can estimate the row (y) gradient across 3 rows

\[
\frac{\partial f}{\partial y} \equiv f_y \approx \frac{1}{3} \left[ \frac{I[x,y+1] - I[x,y-1]}{2} + \frac{I[x-1,y+1] - I[x-1,y-1]}{2} + \frac{I[x+1,y+1] - I[x+1,y-1]}{2} \right]
\]
Gradient from 3x3 neighborhood

Estimate both magnitude and direction of the edge.

\[ f_y = \frac{(38-12) + (66-15)}{2} + \frac{(65-42)}{2} \div 3 = \frac{(13 + 25 + 11)}{3} = 16 \]

\[ f_x = \frac{(65-38) + (64-14) + (42-12)}{2} \div 3 = \frac{(13 + 25 + 15)}{3} = 18 \]

\[ \theta = \tan^{-1} \left( \frac{f_y}{f_x} \right) = \tan^{-1} \left( \frac{16}{18} \right) = 42 \text{ degrees} \]

Sobel mask uses weights of 1,2,1 and -1,-2,-1 in order to give more weight to center estimate. The scaling factor is thus 1/8 and not 1/6.
Computational short cuts

- for magnitude use $\max(\mid \frac{\partial f}{\partial y} \mid, \mid \frac{\partial f}{\partial x} \mid)$
- for magnitude use $\mid \frac{\partial f}{\partial y} \mid + \mid \frac{\partial f}{\partial x} \mid$
- omit direction computation

Alternative masks for gradient

Prewitt: $M_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$ ; $M_y = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$

Sobel: $M_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ ; $M_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$

Roberts: $M_x = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ ; $M_y = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

- Prewitt: gradient of least-squares plane through the 9 pixels
- Sobel: scale of 1/8 easy to compute by shifting
- Roberts: faster because of 2 × 2 neighborhood
- Roberts: gradient computed across diagonals
Computational shortcuts

- Use MAX operation on 1D row and column derivatives.
- Use OR operation on thresholded row and column derivatives.

2 rows of intensity vs difference
Caption for Prewitt image

- (a) Image of Judith Prewitt with two rows selected;
- (b) plot of intensities along selected lower row;
- (c) plot of intensities along selected upper row;
- (d) image of $|f_x| + |f_y|$ using the Prewitt 3x3 operator;
- (e) plot of selected lower row of gradient image;
- (f) plot of selected upper row of gradient image.

Properties of derivative masks

- Coordinates of derivative masks have opposite signs in order to obtain a high response in signal regions of high contrast.
- The sum of coordinates of derivative masks is zero so that a zero response is obtained on constant regions.
- First derivative masks produce high absolute values at points of high contrast.
- Second derivative masks produce zero-crossings at points of high contrast.