Final Sample

CS105: Great Insights in Computer Science
Chris Mansley, Fall 2009
Rules

• Keep at least one empty seat between you and everyone else.

• Please write your answers directly on the pages.

• You may use your notes, your book, a calculator.

• You may **not** discuss the test with anyone else, including online sources (no cell phone, computer).

• You may ask clarification questions during the exam.
1. Exponentially Right?

Which subset of these sentences use the term “exponentially” correctly?

A. Once I got my cell phone, I was **exponentially** better at knowing what time it was.

B. The number of transistors on a chip has increased **exponentially** over the past twenty years.

C. Home prices in the northeast had been rising **exponentially**.

D. My new Mac is **exponentially** more powerful than my previous one.

E. Now that I have kids, my life is **exponentially** more complex.
Consider the word “valuable”. If we repeatedly pick a random letter from the word...

A. How many times, on average, do we pick before we get the letter “b”?

B. How many times, on average, do we pick before getting one of the letters in the word “vat”?

C. How many times, on average, do we pick before choosing a vowel?
Find a Huffman code for “lolfootball”. How many bits does it take to write this string in your code?
4. Song Growth

How many syllables do you sing in $n$ verses of the following three songs? Choose from $O(n)$, $O(n \lg n)$, $O(n^2)$, and $O(n^2 \lg n)$.

Verses are numbered.

A.

B.

C.
1. A kitty and a cat with mittens brand new got into an elevator (top floor). Then there were two: 1, 2!

2. Two little animals feeling carefree. A bull came from market. Then there were three: 1, 2, 3!

3. Three little animals waiting to arrive. In came a mule and a mare. And then there were five: 1, 2, 3, 4, 5!

4. Five little animals acting sedate. The Doggy family joined them. And then there were eight: 1, 2, 3, 4, 5, 6, 7, 8!

5. Eight little animals standing in a line. Shep the sheep sauntered in. And then there were nine: 1, 2, 3, 4, 5, 6, 7, 8, 9!

6. Nine little animals waiting yet again. Mr. Hog got on. And then there were ten: 1, 2, 3, 4, 5, 6, 7, 8, 9, one oh!

... (From Sesame Street. Assume for your analysis that between one and three animals get on per floor.)
4b. There's A Hole

1. **There's a hole** at the bottom of the sea. There's a hole at the bottom of the sea. There's a hole, there's a hole, there's a hole at the bottom of the sea.

2. **There's a log** in the hole at the bottom of the sea. There's a log in the hole at the bottom of the sea. There's a log, there's a log, there's a log in the hole at the bottom of the sea.

3. **There's a bump** on the log in the hole at the bottom of the sea. There's a bump on the log in the hole at the bottom of the sea. There's a bump, there's a bump, there's a bump on the log in the hole at the bottom of the sea.

4. **There's a frog** on the bump on the log in the hole at the bottom of the sea. There's a frog on the bump on the log in the hole at the bottom of the sea. There's a frog, there's a frog, there's a frog on the bump on the log in the hole at the bottom of the sea.

5. **There's a leg** on the frog on the bump on the log in the hole at the bottom of the sea. There's a leg on the frog on the bump on the log in the hole at the bottom of the sea. There's a leg, there's a leg, there's a leg on the frog on the bump on the log in the hole at the bottom of the sea.

...
The ants go marching one by one, hurrah, hurrah
The ants go marching one by one, hurrah, hurrah
The ants go marching one by one,
The little one stops to suck his thumb
And they all go marching down to the ground
To get out of the rain, BOOM! BOOM! BOOM!

The ants go marching two by two, hurrah, hurrah
The ants go marching two by two, hurrah, hurrah
The ants go marching two by two,
The little one stops to tie his shoe
And they all go marching down to the ground
To get out of the rain, BOOM! BOOM! BOOM!

The ants go marching three by three, hurrah, hurrah
The ants go marching three by three, hurrah, hurrah
The ants go marching three by three,
The little one stops to climb a tree
And they all go marching down to the ground
To get out of the rain, BOOM! BOOM! BOOM!

The ants go marching four by four, hurrah, hurrah
The ants go marching four by four, hurrah, hurrah
The ants go marching four by four,
The little one stops to shut the door
And they all go marching down to the ground
To get out of the rain, BOOM! BOOM! BOOM!

The ants go marching five by five, hurrah, hurrah
The ants go marching five by five, hurrah, hurrah
The ants go marching five by five,
The little one stops to take a dive
And they all go marching down to the ground
To get out of the rain, BOOM! BOOM! BOOM!

The ants go marching six by six, hurrah, hurrah
The ants go marching six by six, hurrah, hurrah
The ants go marching six by six,
The little one stops to pick up sticks
And they all go marching down to the ground
To get out of the rain, BOOM! BOOM! BOOM!

The ants go marching seven by seven, hurrah, hurrah
The ants go marching seven by seven, hurrah, hurrah
The ants go marching seven by seven,
The little one stops to pray to heaven
And they all go marching down to the ground
To get out of the rain, BOOM! BOOM! BOOM!

The ants go marching eight by eight, hurrah, hurrah
The ants go marching eight by eight, hurrah, hurrah
The ants go marching eight by eight,
The little one stops to shut the gate
And they all go marching down to the ground
To get out of the rain, BOOM! BOOM! BOOM!
A. What’s the shortest path from G to F?

B. What’s a longest (cycle-free) path from G to C?
The Scratch scripts below work on the 1 through $n$ elements of list $L$ and say either True or False. Match each script to the question it answers.

A. Does the number -1 appear in the list?

B. Are all elements in the list ten or larger?

C. Is there an element in the list whose position matches the number (like the number 5 in the 5th position)?
The Scripts

when I receive mystery1
set index to 1
repeat length of L
if item index of L < 10
say No
stop script
change index by 1
say Yes
stop script

when I receive mystery2
set index to 1
repeat length of L
if item index of L = 1
say Yes
stop script
change index by 1
say No
stop script

when I receive mystery3
set index to 1
repeat length of L
if item index of L = index
say Yes
stop script
change index by 1
say No
stop script
7. Unsorted Algorithms

You have a list of integers of length \( n \). How fast can you answer the following questions if the list is unsorted? Your choices are \( O(1) \), \( O(\log n) \), \( O(n) \), \( O(n^2) \).

A. What is the second biggest number in the list?

B. Are there two numbers in the list that add up to 1000?

C. How many times does 74 appear in the list?

D. Is the product of the numbers divisible by 7?
8. Sorted Algorithms

You have a list of integers of length $n$. How fast can you answer the following questions if the list is sorted? Your choices are $O(1), O(\log n), O(n), O(n^2)$.

A. What is the second biggest number in the list?

B. Are there two numbers in the list that add up to 1000?

C. How many times does 74 appear in the list?

D. Is the product of the numbers divisible by 7?
A. Is the product of these numbers divisible by 5?

B. Is the median of these 51 numbers bigger than 500?
For each of the following evaluation trees, write down the corresponding Boolean formula that is being evaluated:

A.
\[
\text{or} \quad \text{and} \quad \text{not}
\]

B.
\[
\text{and} \quad \text{or}
\]

- A.
  \[
  \text{or} \quad \text{and} \quad \text{not}
  \]

- B.
  \[
  \text{and} \quad \text{or}
  \]
A. Use the tree on the next page to find a Huffman encoding for the word “please”. (Reminder: We use the convention that the top branch is 0 and the lower branch is 1)

B. What does 10000100111010 spell?
Write a logical expression with A, B, C, and, or, not, as needed.

\[
D = \text{A \ \land \ \text{B \ \land \ \text{C}}}
\]
Barcodes are a great example of error correcting codes. Correct the codes below by providing the missing digit.

A. 024543143x05

B. 7940x1293824

C. 02454306951x
What will the sprite say when clicked?
For what values of “?” does each of these scratch scripts halt?
What will \( E \) be after each of these short machine-language programs are executed?

A.  
\[
\begin{align*}
\text{acc} &= \text{not } A \\
\text{acc} &= \text{acc and } B \\
\text{acc} &= \text{acc or } C \\
E &= \text{acc}
\end{align*}
\]

B.  
\[
\begin{align*}
\text{acc} &= \text{B} \\
\text{acc} &= \text{acc and } A \\
E &= \text{acc} \\
\text{acc} &= \text{not } B \\
\text{acc} &= \text{acc and } C \\
\text{acc} &= \text{acc or } E \\
E &= \text{acc}
\end{align*}
\]
A. List the nodes reachable from A.

B. List the source(s).
A $k$-exactly-one gate outputs True if exactly one of its $k$ input bits are True. It can be made with one AND gate, one $k$-at-most-one gate, and one $k$-or gate.

A $k$-at-most-one gate outputs True if no more than one of its $k$ input bits are True. It can be made with $k(k-1)/2$ AND gates, one NOT gate, and one $k$-or gate.

A $k$-or gate outputs True if at least one of its $k$ input bits are True. It is made of $k-1$ OR gates.

How many AND gates are there in a 10-exactly-one gate?
19. Find The Bug

The number of described U.S. species of beetles is 23,700. If I’m thinking of one of them, how many guesses would it take for you to figure out which, using only yes/no questions?
20. The Heat Is On

Both scripts have bugs. Choose one description for each.

A. Turns on the heat when it’s hot.

B. Turns on the heat at most once.

C. Leaves the heat always on or always off.

D. Turns off the heat at most once.

E. Never turns on the heat, no matter what.