Chapter 5: Algorithms and Heuristics

CS105: Great Insights in Computer Science
 Algorithms

- sockA, sockB, and sockC represent different approaches to solving the problem of sock sorting.
- A concrete approach to solving a problem is called an algorithm.
- Different algorithms can be better or worse in different ways.
Lessons Learned

• Given a notion of “time” (# of socks removed or # of statements executed), can compare different algorithms based on the time they take.

• They really are different, so use good algorithms.

• I once redesigned a colleague’s algorithm and it ran in seconds where it used to take an hour.

• Hard to believe they solved the same problem.
You Think You Have Problems

- The sock matching problem is a little silly, because the right answer seems sort of obvious.

- Algorithms researchers pride themselves on finding simple-sounding problems for which the best algorithm is not obvious.

- Ah, but what’s a problem? It should have a well-defined input and a well-defined output.
A natural class of problems is decision problems on lists of numbers. That is, the input is a list of numbers and the output is a decision, essentially a bit (True or False, yes or no).

We’ll try some examples and see if we can find efficient algorithms for solving different problems.
List Membership

- Problem: "Is $x$ in the list"? Given a list of numbers and a number $x$, is the number $x$ in the list?

- Is 83 in the list?
List Membership

• Problem: “Is $x$ in the list”? Given a list of numbers and a number $x$, is the number $x$ in the list?

• Is 83 in the list?

48, 40, 14, 46, 31, 0, 27, 12, 22, 71, 45, 63, 30, 64, 83, 28, 97, 90, 85, 52
Loop on List

- Runs through list.
- Checks if each element equals \( x \).
- If it finds one, says “yes” right away.
- If it reaches the end of the list, says “no”.

![Scratch code](image)
Problem: “Is the sum divisible by 5”? Given a list of numbers, is the sum of the numbers divisible by 5?

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Is the sum divisible by 5?

48, 40, 14, 46, 31, 0, 27, 12, 22, 71, 45, 63, 30, 64, 83, 28, 90, 85, 52
Direct Method

- i points to each element of the list in turn
- total keeps the running sum
- afterwards, test divisibility by 5
Finding the Max

- Problem: “Is the max $x$”? Given a list of numbers and a number $x$, is the largest number in the list equal to $x$?
- Is the max 98?
Finding the Max

• Problem: “Is the max $x$”? Given a list of numbers and a number $x$, is the largest number in the list equal to $x$?

• Is the max 98?

48, 40, 14, 46, 31, 0, 27, 12, 22, 71, 45, 63, 30, 64, 83, 28, 97, 90, 85, 52
Specifying an algorithm to the computer is like talking to a little kid. You need to spell out everything precisely.

Let’s try.
Problem: “Is the product divisible by 5”? Given a list of numbers, is the product of the numbers divisible by 5?

Is the product divisible by 5?
Product Divisible By 5?

• Problem: “Is the product divisible by 5”? Given a list of numbers, is the product of the numbers divisible by 5?

• Is the product divisible by 5?

48, 41, 14, 46, 31, 2, 27, 12, 22, 71, 44, 63, 33, 64, 83, 28, 96, 87, 52
Anatomy of a Loop

- Set initial conditions.
- Run through all elements (i is points to each of the elements).
- Update the value.
- Report the answer.
Finding the Median

- Problem: “Is the median $x$”? Given a list of numbers and a number $x$, is the median number in the list equal to $x$?

- Is the median 45?
Problem: “Is the median $x$”? Given a list of numbers and a number $x$, is the median number in the list equal to $x$?

Is the median 45?

48, 40, 14, 46, 31, 0, 27, 12, 22, 71, 45, 63, 30, 64, 83, 28, 90, 85, 52
Check the Median

- Assuming we can sort (soon!)...

- Sort, then check the middle item.

- Sorting is relatively slow (try it).
• If there are 19 numbers, median is the one in the 10th position when the list is sorted.

• That’s fine, but, there’s a shortcut: (1) count the number of numbers less than $x$ ($L$) and the number equal to $x$ ($E$), (2) check if $L < 10$ and $L+E > 9$.

• In general form, $L < (n+1)/2$ and $L+E > (n-1)/2$. 
How Do It Fast? Median

- No sorting necessary.
- Let’s try it! Is the median 47?
How Do It Fast? Median

- No sorting necessary.
- Let’s try it! Is the median 47?

9, 13, 2, 24, 13, 60, 31, 62, 59, 70, 47, 47, 77, 46, 46, 70, 39, 6, 63
How Do It Fast? Sum

• Straightforward algorithm is: (1) sum up the numbers in the list, (2) divide the grand total by 5, (3) check if there’s no remainder.

• There are a few shortcuts we can use: (A) A number is divisible by 5 if and only if it ends in 5 or 0. (B) The sum of two numbers is divisible by 5 only if the sum of their last digits is divisible by 5.

• Faster algorithm: (1) Keep a running total of the last digits, keeping only the last digit of the sum, (2) check if it’s 0 or 5. Let’s try: Is the sum divisible by 5?
How Do It Fast?  Sum

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9, 13, 2, 24, 13, 60, 31, 62, 59, 70, 47, 47, 77, 46, 46, 70, 39, 6, 63
How Do It Fast? Product

• Straightforward algorithm is: (1) multiply all the numbers in the list, (2) divide the product by 5, (3) check if there’s no remainder.

• There are another shortcut we can use: A product is divisible by 5 if and only if at least one of the multiplicands is divisible by 5.

• Faster algorithm: Check if any number in the list ends with 0 or 5. Let’s try. Is the product divisible by 5?
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• There are another shortcut we can use: A product is divisible by 5 if and only if at least one of the multiplicands is divisible by 5.

• Faster algorithm: Check if any number in the list ends with 0 or 5. Let’s try. Is the product divisible by 5?

9, 13, 2, 24, 13, 61, 31, 62, 59, 72, 47, 45, 77, 46, 46, 73, 39, 6, 63
Comparing Algorithms

• Next, we’ll talk about how computer scientists compare algorithms to decide which is better.

• Could try them all, but that would defeat the purpose, wouldn’t it?

• But first, an aside to introduce some mathematical concepts.