Chapter 4: How Universal Are Turing Machines?

CS105: Great Insights in Computer Science
Quicksort

- quicksort(list):
  - if len of list <= 1: return
  - [list1, list2] = partition(list)
  - quicksort(list1)
  - quicksort(list2)
**Quicksort cont...**

<table>
<thead>
<tr>
<th>List</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
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<tr>
<td>3</td>
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<td>4</td>
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<tr>
<td>6</td>
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<tr>
<td>9</td>
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</tbody>
</table>
Quicksort cont...

List:

7 3 8 2 5 1 4 6 9
Quicksort cont...

List

| 7 | 3 | 8 | 2 | 5 | 1 | 4 | 6 | 9 |

List1

| 3 | 2 | 1 | 4 |
Quicksort cont...

List

7 3 8 2 5 1 4 6 9

List1

3 2 1 4

List2

7 8 6 9
Quicksort cont...
Quicksort cont...

List

7 3 8 2 5 1 4 6 9

List1

3 2 1 4

List2

7 8 6 9

List1

1
Quicksort cont...
Quicksort cont...

List

7 3 8 2 5 1 4 6 9

List1

3 2 1 4

List2

7 8 6 9

List1

1

List2

3 4
Quicksort cont...

List:

7 3 8 2 5 1 4 6 9

List1:

3 2 1 4

List2:

7 8 6 9

List1:

1

List2:

3 4

List1:

1

List1:
Quicksort cont...

List
[7, 3, 8, 2, 5, 1, 4, 6, 9]

List1
[3, 2, 1, 4]

List2
[7, 8, 6, 9]
Quicksort cont...

List

7 3 8 2 5 1 4 6 9

List1

3 2 1 4

List2

7 8 6 9

List1

1

List2

3 4

List1

1

List2

3 4

List1

4
Quicksort cont...

List

7  3  8  2  5  1  4  6  9

List1

3  2  1  4

List2

7  8  6  9

List1

1  3  4

List2

7  6

List1

1  3  4

List2

7  6
Quicksort cont...

List

7 3 8 2 5 1 4 6 9

List1

3 2 1 4

List2

7 8 6 9

List1

1

List2

3 4

List1

7 6

List2

9
Quicksort cont...

List

7 3 8 2 5 1 4 6 9

List1

3 2 1 4

List2

7 8 6 9

List1

1

List2

3 4

List1

7 6

List2

9

List1

4
Quicksort cont...
Quicksort cont...

List

7 3 8 2 5 1 4 6 9

List1

3 2 1 4

List2

7 8 6 9

List1

1

List2

3 4

List1

7 6

List2

9
Church-Turing Thesis

• Usually taken to mean that any machine computation can be carried out on a Turing machine.

• Sometimes expanded to mean that any physical system can be simulated on a Turing machine.

• And, since brains are physical systems, our minds must be equivalent to Turing machines!
Quantum Effects

- Pseudo-random-based encryption always has a chance of being cracked.
- Only source of true randomness: quantum mechanics (the rest of physics is deterministic, if chaotic).
- Einstein didn’t like it. Tough.
Quantum Randomness

Schrodinger's Cat

1. Radioactive material has a 50:50 chance of triggering the Geiger counter.

2. If the Geiger counter is triggered, the hammer falls.

3. The hammer breaks the poison bottle.

4. The cat dies if the poison bottle breaks.

5. The cat lives if the Geiger counter does not trigger the hammer and releases the poison.
Quantum Computer

- A quantum bit (qubit) is simultaneously zero and one (a superposition). $n$ qubits can represent $2^n$ possibilities.

- When you look, one possibility presents itself, according to well understood probabilistic rules. A kind of parallel search.

- **Shor**: A computer with qubits can factor numbers in polynomial time!
If Factoring is Easy...

- quantum computers invalidate standard cryptosystems. *No more secrets.*

- However, they also open up some wild possibilities.

- *quantum cryptography:* qubits can be completely random and correlated at a distance. Can be used to send absolutely secret messages.
QUANTUM MECHANICS HIDES A SECRET CODE KEY

Alice and Bob try to keep a quantum-cryptographic key secret by transmitting light in the form of polarized photons, a scheme invented by Charles Bennett of IBM and Gilles Brassard of the University of Montreal during the 1980s and now implemented in a number of commercial products.

1. To begin creating a key, Alice sends a photon through either the 0 or 1 slot of the rectilinear or diagonal polarizing filters, while making a record of the various orientations.

2. For each incoming bit, Bob chooses randomly which filters he uses for detection and writes down both the polarization and the bit value.

3. If Eve the eavesdropper tries to spy on the train of photons, quantum mechanics prohibits her from using both filters to detect the orientation of a photon. If she chooses the wrong filter, she may create errors by modifying their polarization.

4. After all the photons have reached Bob, he tells Alice over a public channel, perhaps by telephone or e-mail, the sequence of filters he used for the incoming photons, but not the bit value of the photons.

5. Alice tells Bob during the same conversation which filters she chose correctly. These instances constitute the bits that Alice and Bob will use to form the key that they will use to encrypt messages.
Today’s Advice

- My only piece of advice for today:
- Don’t listen to anything I say today; it will hurt your head.
The main topic today is self reference and self contradiction.

The idea is that "interesting" things happen when something can refer to itself and assert that it has properties that negate its own existence.
Oxymorons

• We’re all familiar with oxymorons: words that harbor two conflicting meanings.

• Top ten list from http://www.oxymoronlist.com/:
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- 2. Healthy Tan
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  8. Working Vacation
  7. Tax Return
  6. Virtual Reality
  5. Dodge Ram
  4. Work Party
  3. Jumbo Shrimp
  2. Healthy Tan
  1. Microsoft Works
Surface Contradiction

- Examples seem incongruous, but they all actually make sense.
  - “jumbo shrimp” just means pretty big for a shrimp, which makes perfect sense.
- The contradiction isn’t very deep.
You Don’t Say...

- We're not f-ing bitter!
- I am not in denial.
- Eschew obfuscation.
- When you least expect it, expect it.
- But, I don't speak English.
- I’m not talking to you.
- I can’t talk right now.
- I’m sorry, but I just don’t care.
- Leave me be, I’m asleep.
- I’m speechless.
- I prefer not to have an opinion.
- As a general rule, we handle everything on a case-by-case basis.
- I told you he’s unpredictable!
- OK is “oll korrect” (1939)!
- Don’t let me pressure you!
• Know what I hate most? Rhetorical questions. (Camp)

• They all laughed when I said I was going to be a comedian. Well, they're not laughing now. (Munkhouse)

• If there is anything the nonconformist hates worse than a conformist, it's another nonconformist who doesn't conform to the prevailing standard of nonconformity. (Vaughan)

• You know what I hate? Indian givers...no, I take that back. (Phillips)

• Last month I blew $5,000 on a reincarnation seminar. I figured, hey, you only live once. (Shakes)

• I almost had a psychic girlfriend but she left me before we met. Plan to be spontaneous tomorrow. I'd kill for a Nobel Peace Prize. I was trying to daydream, but my mind kept wandering. (Wright)
Untitled by Anonymous

There is nothing which cannot be improved

KEEP DOOR CLOSED AT ALL TIMES

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/* no comment */

Leci, n’est pas une pipe.
Escher Parody
There’s a weird idea here: You shouldn’t be able to create a statement that, if interpreted properly, results in another statement that contradicts the original statement.

It’s a “go back in time and kill your own grandfather” sort of thing.

There are a bunch of deep mathematical insights that come from applying this idea.

Here’s a quick survey before the main event.
Russell’s Paradox

- In the town of Chelm, there’s a barber. The barber is required to shave every man in town who does not shave himself.

- Who shaves the barber?

- For the statement to be true:
  - If he shaves himself, then he does not need to shave himself.
  - If he does not shave himself, then he needs to shave himself.
This example was created to show that there are limitations to how you can create meaningful sets.

If there is unrestricted self-reference, you can create impossible situations.
Gödel’s Theorem

“This statement cannot be proven.”

- Kurt Gödel showed that any system of mathematics that includes the integers can express this self-referential statement.
  - If it’s true, you can’t prove it! (Incompleteness.)
  - If it’s false, you can prove something that’s false! (Inconsistency.)
- Tough choice.
Cantor’s Diagonalisation

• How many fractions are there? Infinite.
• How many decimals are there? Infinite.
• Are they the same size infinity?
• Well, we can make an infinitely long list that includes every fraction:
### List of Fractions

- Start with all the fractions where the numerator and denominator add up to 1, then 2, then 3.

<p>| | | |</p>
<table>
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</tbody>
</table>
• Can we list all the decimals?
• The “add to a constant” trick doesn’t work anymore, since we have decimals like 0.3333... where the digit sum is infinite.
• So, let’s say we can list them all.
• Here’s the list, hypothetically:
On The List?

- Read down the diagonal.
  0.5898032467...
- Add 1 to each digit (with wraparound).
  0.6909143578...
- The resulting decimal is not on the list! (Differs from the $i$th one in the $i$th digit.)

0.57953916570123654
0.98877675309679680
0.54921087722147810
0.97889400202076116
0.68930952976230064
0.73758318399567813
0.33201823212447767
0.62085164445848273
0.22859580612307950
0.26912510570620329

...
On The List?

• Read down the diagonal.
  0.5898032467...

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  0.6909143578...

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...
Conclusion

• You can’t make a list of all the decimals.
• You can make a list of all the fractions.
• There are more decimals (real numbers) than fractions (rational numbers)!