

Discrete Shells Simulation

Xiaofeng Mi

This project is an implementation of Grinspun's discrete shells, the model of which is governed by nonlinear membrane and flexural energies. These energies measure differences between the undeformed configuration and the deformed one. The energies are invariant under rigid body transformation of both the deformed and the undeformed shape and it's simple to implement as far as the steps one need to take is concerned. The hardest part of the implementation turns out to be the correctly deriving of the derivatives and building the linear system for implicit solver. So the remainder of the report will be focusing on revealing the derivative details which the original paper did not touch. Actually, Eitan Grinspun, the author of the discrete shells paper, posted an automatic derivatives package for use to get derivatives by program, unfortunately, one of my purposes of playing the discrete shells simulation is to practice deriving complex math, and to playing with vector, matrices and tensors by hand, so I did not follow the original authors' way.

Basic Framework

Instead of following Grinspun's paper directly, the implementation follows Baraff's work on cloth simulation. I did not use energies directly, instead, I Define behavior function $C(x_1, \dots, x_n)$ from X to R^m , which should be zero under resting state, that is, when the shells are in the original configurations, or, say, are happy!

$$\begin{cases} \mathbf{f} = m\ddot{\mathbf{x}} \\ \dot{\mathbf{v}} = \ddot{\mathbf{x}} = \mathbf{f} / m \\ \dot{\mathbf{x}} = \mathbf{v} \end{cases}$$

$$\text{or } \begin{pmatrix} \Delta \mathbf{x} \\ \Delta \mathbf{v} \end{pmatrix} = h \begin{pmatrix} \mathbf{v}_0 \\ \mathbf{M}^{-1} \mathbf{f}_0 \end{pmatrix}$$

$$E = \frac{k_s}{2} C^T C$$

$$f_i = -\frac{\partial E}{\partial x_i} = -k_s C \frac{\partial C}{\partial x_i}$$

With damping force

And just like Baraff's paper did, I modeled damping force explicitly because I use backward Euler to solve the ODE system. Grinspun used implicit Newmark and embed the damping force inherently, which I believe is a better way to do.

$$f_i = -\frac{\partial C}{\partial P_i} (k_s C + k_d \cdot \dot{C}) = -\frac{\partial C}{\partial P_i} (k_s \cdot C + k_d \left(\frac{\partial C}{\partial P_i} \right)^T \cdot v)$$

$$\frac{\partial f_i}{\partial P_j} = -k_s \frac{\partial^2 C}{\partial P_i \partial P_j} \cdot C - k_s \frac{\partial C}{\partial P_i} \left(\frac{\partial C}{\partial P_j} \right)^T - k_d \frac{\partial^2 C}{\partial P_i \partial P_j} \cdot \dot{C} - k_d \frac{\partial C}{\partial P_i} \left(\frac{\partial \dot{C}}{\partial P_j} \right)^T$$

$$= -k_s \frac{\partial C}{\partial P_i} \left(\frac{\partial C}{\partial P_j} \right)^T - \frac{\partial^2 C}{\partial P_i \partial P_j} (k_s C + k_d \dot{C})$$

$$\frac{\partial f_i}{\partial v_j} = -\frac{\partial C}{\partial P_i} \cdot k_d \cdot \left(\frac{\partial C}{\partial P_j} \right)^T = -k_d \frac{\partial C}{\partial P_i} \left(\frac{\partial C}{\partial P_j} \right)^T$$

Grinspun's model

Following Grinspun's model, the behavior functions would be:

$$C_L = \sqrt{\|\bar{e}\|} \left(1 - \frac{\|e\|}{\|\bar{e}\|}\right) \quad \text{to keep edge length}$$

$$C_A = \sqrt{\|\bar{A}\|} \left(1 - \frac{\|A\|}{\|\bar{A}\|}\right) \quad \text{to keep face area}$$

$$C_B = \sqrt{\frac{\|\bar{e}\|}{h_e}} (\theta_e - \bar{\theta}_e) \quad \text{to keep surface curvature}$$

And just following Baraff's work for the implicit system derivation:

$$\begin{pmatrix} \Delta \mathbf{x} \\ \Delta \mathbf{v} \end{pmatrix} = h \begin{pmatrix} \mathbf{v}_0 \\ \mathbf{M}^{-1} \mathbf{f}_0 \end{pmatrix} = h \begin{pmatrix} \mathbf{v}_0 \\ \mathbf{M}^{-1} \left(\mathbf{f}_0 + \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \Delta \mathbf{x} + \frac{\partial \mathbf{f}}{\partial \mathbf{v}} \Delta \mathbf{v} \right) \end{pmatrix}$$

$$\Delta \mathbf{v} = h \mathbf{M}^{-1} \left(\mathbf{f}_0 + \frac{\partial \mathbf{f}}{\partial \mathbf{x}} h (\mathbf{v}_0 + \Delta \mathbf{v}) + \frac{\partial \mathbf{f}}{\partial \mathbf{v}} \Delta \mathbf{v} \right)$$

$$\left(\mathbf{I} - h \mathbf{M}^{-1} \frac{\partial \mathbf{f}}{\partial \mathbf{x}} - h^2 \mathbf{M}^{-1} \frac{\partial \mathbf{f}}{\partial \mathbf{v}} \right) \Delta \mathbf{v} = h \mathbf{M}^{-1} \left(\mathbf{f}_0 + h \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \mathbf{v}_0 \right)$$

In my implementation, I use the standard pre-conditioned bi-conjugate gradient method to solve the sparse linear system. Baraff used his own modified conjugate gradient – it doesn't matter – in the implementation, solving the sparse linear system is quick compared with the effort of computing the system out.

Some Notations

Before proceed to the actual derivatives (which will not be interesting at all!), I would like to introduce some notations to make the derivatives clean. If you have read Dean P. Macri or David Pritchard's write up for implementing Baraff's cloth simulation, you will find that the derivations are huge and scaring. At least I am very scared! But I did get a lot help from these works and worked out mine for the shells model.

I followed their way to use the skew-symmetric transformation. And I extended the operator to transform a matrix to a tensor, and they have a lot of interesting beautiful properties, just like other math!

$$\text{Define } \llbracket w \rrbracket = \begin{pmatrix} 0 & -w_z & w_y \\ w_z & 0 & -w_x \\ -w_y & w_x & 0 \end{pmatrix}, \text{ where } w \text{ is a vector in } R^3, \text{ and } \llbracket w \rrbracket \text{ is a } 3 \times 3 \text{ matrix.}$$

Suppose $M = (v_1 \ v_2 \ v_3)$, where v_i is a vector in R^3 ,

$$\text{Define } \llbracket M \rrbracket = \begin{pmatrix} \llbracket v_1 \rrbracket \\ \llbracket v_2 \rrbracket \\ \llbracket v_3 \rrbracket \end{pmatrix}, \text{ that is, } \llbracket M \rrbracket \text{ is a } 3 \times 3 \times 3 \text{ tensor.}$$

Note that the $\llbracket \cdot \rrbracket$ to transform a vector to a skew-symmetric matrix is a standard operator, while to transform a matrix to a tensor is not.

Let's see some useful properties of the skew-symmetric operators so that we can avoid the expressions from going too large and crazy.

Property 1. $\llbracket v_1 \rrbracket v_2 = v_1 \times v_2$

It follows that:

Property 2. $\llbracket v_1 \rrbracket v_2 = -\llbracket v_2 \rrbracket v_1$

Just use the fact that $v_1 \times v_2 = -v_2 \times v_1$ to prove this property.

Property 3. $\llbracket M \rrbracket v = -\llbracket v \rrbracket M$

Proof: let $M = \begin{pmatrix} | & | & | \\ m_1 & m_2 & m_3 \\ | & | & | \end{pmatrix}$, then

$$\llbracket M \rrbracket v = \begin{pmatrix} | & | & | \\ \llbracket m_1 \rrbracket v & \llbracket m_2 \rrbracket v & \llbracket m_3 \rrbracket v \\ | & | & | \end{pmatrix} = \begin{pmatrix} | & | & | \\ -\llbracket v \rrbracket m_1 & -\llbracket v \rrbracket m_2 & -\llbracket v \rrbracket m_3 \\ | & | & | \end{pmatrix} = -\llbracket v \rrbracket M$$

From the second property, we can easily find that

Property 4. $\llbracket \llbracket v_1 \rrbracket \rrbracket v_2 = -\llbracket v_2 \rrbracket \llbracket v_1 \rrbracket$

Property 5. $\llbracket I \rrbracket v = -\llbracket v \rrbracket$

And also some useful derivative properties related to the $\llbracket \bullet \rrbracket$ operator:

$$\text{Property 6. } \frac{\partial \llbracket v_1 \rrbracket}{\partial v_2} = \begin{pmatrix} \frac{\partial \llbracket v_1 \rrbracket}{\partial v_{2x}} \\ \frac{\partial \llbracket v_1 \rrbracket}{\partial v_{2y}} \\ \frac{\partial \llbracket v_1 \rrbracket}{\partial v_{2z}} \end{pmatrix} = \begin{pmatrix} \llbracket \frac{\partial v_1}{\partial v_{2x}} \rrbracket \\ \llbracket \frac{\partial v_1}{\partial v_{2y}} \rrbracket \\ \llbracket \frac{\partial v_1}{\partial v_{2z}} \rrbracket \end{pmatrix} = \llbracket \frac{\partial v_1}{\partial v_2} \rrbracket$$

Here $\frac{\partial v_1}{\partial v_2}$ is just a standard Jacobian.

Property 7.

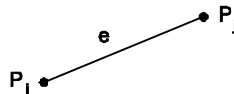
$$\frac{\partial(v_1 \times v_2)}{\partial v_3} = \frac{\partial(\llbracket v_1 \rrbracket v_2)}{\partial v_3} = \llbracket \frac{\partial v_1}{\partial v_3} \rrbracket v_2 + \llbracket v_1 \rrbracket \frac{\partial v_2}{\partial v_3} = \llbracket \frac{\partial v_1}{\partial v_3} \rrbracket v_2 - \llbracket \frac{\partial v_2}{\partial v_3} \rrbracket v_1 = \llbracket v_1 \rrbracket \frac{\partial v_2}{\partial v_3} - \llbracket v_2 \rrbracket \frac{\partial v_1}{\partial v_3}$$

All these properties will be used widely to simplify the notations throughout the following derivations.

Forces and their derivatives

As described in the Grinspun's paper, there are two type of energies: the membrane energy and the flexural energy, and totally we have two types of membrane force: the one to keep length and the one to keep area. And for the flexural energy, the only type of force is to keep extrinsic curvatures.

To keep length constant



$$C_L = \sqrt{\|e\|} \left(1 - \frac{\|e\|}{\|e\|}\right)$$

$$\begin{aligned} \frac{\partial C_L}{\partial P_i} &= -\sqrt{\|e\|} \cdot \frac{1}{\|e\|} \cdot \frac{\partial \|e\|}{\partial P_i} = -\frac{1}{\sqrt{\|e\|}} \cdot \frac{\partial ((P_i - P_j)^T (P_i - P_j))^{\frac{1}{2}}}{\partial P_i} = -\frac{1}{\sqrt{\|e\|}} \cdot \frac{P_i - P_j}{\|e\|} \\ &= \frac{1}{\sqrt{\|e\|}} \cdot \frac{P_j - P_i}{\|e\|} \end{aligned}$$

Note that here, the force is:

$$F_i = -k_s \cdot \sqrt{\|\bar{e}\|} \left(1 - \frac{\|e\|}{\|\bar{e}\|}\right) \cdot \frac{1}{\sqrt{\|\bar{e}\|}} \cdot \frac{P_j - P_i}{\|e\|} = k_s \frac{(\|e\| - \|\bar{e}\|)(P_j - P_i)}{\|\bar{e}\| \|e\|}$$

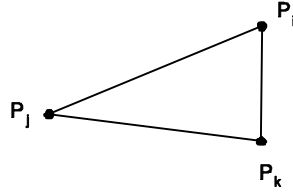
And notice that $(P_j - P_i)/\|e\|$ is a directional unit vector, which make the length model exactly a spring system model.

$$\frac{\partial^2 C_L}{\partial P_i^2} = -\frac{I_{3 \times 3}}{\sqrt{\|\bar{e}\|} \|e\|}$$

$$\frac{\partial^2 C_L}{\partial P_i \partial P_j} = \frac{I_{3 \times 3}}{\sqrt{\|\bar{e}\|} \|e\|}$$

To keep area constant

The derivatives related with area forces are given here, but in my implementation, I skipped this part because I found that for a triangular mesh system, the force model which to keep the lengths constant suffices.



$C_A = \sqrt{\|\bar{A}\|} \left(1 - \frac{\|A\|}{\|\bar{A}\|}\right)$, where $\|A\| = \frac{1}{2}(\bar{A}^T \bar{A})^{\frac{1}{2}}$, and $\|\bar{A}\| = 2\|A\|$, the notation is pretty strange, but

anyway. $\|A\|$ is the area of the triangle, and \bar{A} is the cross product of two adjacent edge vectors.

$$\frac{\partial C_A}{\partial P_i} = -\frac{1}{\sqrt{\|\bar{A}\|}} \cdot \frac{\partial \|A\|}{\partial P_i}$$

$$\frac{\partial \|A\|}{\partial P_i} = \frac{1}{4\sqrt{\|A\|}} \cdot 2 \cdot \frac{\partial \bar{A}}{\partial P_i} \cdot \bar{A} = \frac{1}{2\sqrt{\|A\|}} \cdot \frac{\partial \bar{A}}{\partial P_i} \cdot \bar{A}$$

$$\frac{\partial \bar{A}}{\partial P_i} = \frac{\partial((P_j - P_i) \times (P_k - P_i))}{\partial P_i} = -\llbracket(P_k - P_j)\rrbracket \frac{\partial(P_j - P_i)}{\partial P_i} = -\llbracket(P_j - P_k)\rrbracket$$

$$\frac{\partial C_A}{\partial P_i} = -\frac{1}{\sqrt{\|\bar{A}\|}} \cdot \frac{1}{2\|A\|} \cdot \llbracket(P_k - P_j)\rrbracket \bar{A} = \frac{1}{2\sqrt{\|\bar{A}\|} \|A\|} \cdot (P_j - P_k) \times \bar{A}$$

$$\frac{\partial((P_j - P_k) \times \bar{A})}{\partial P_i} = \frac{\partial(\llbracket(P_j - P_k)\rrbracket \bar{A})}{\partial P_i} = \llbracket(P_j - P_k)\rrbracket \left(\frac{\partial \bar{A}}{\partial P_i}\right) = -\llbracket(P_j - P_k)\rrbracket^2$$

$$\frac{\partial((P_j - P_k) \times \bar{A})}{\partial P_j} = \llbracket(P_j - P_k)\rrbracket \frac{\partial \bar{A}}{\partial P_j} - \llbracket\bar{A}\rrbracket I = -\llbracket(P_j - P_k)\rrbracket \llbracket(P_k - P_i)\rrbracket - \llbracket\bar{A}\rrbracket$$

$$\frac{\partial((P_j - P_k) \times \bar{A})}{\partial P_k} = \llbracket(P_j - P_k)\rrbracket \frac{\partial \bar{A}}{\partial P_k} - \llbracket\bar{A}\rrbracket (-I) = -\llbracket(P_j - P_k)\rrbracket \llbracket(P_i - P_j)\rrbracket + \llbracket\bar{A}\rrbracket$$

Let $a = \frac{1}{2\sqrt{\|\bar{A}\|} \|A\|}$, we have all the terms in evaluating the derivatives:

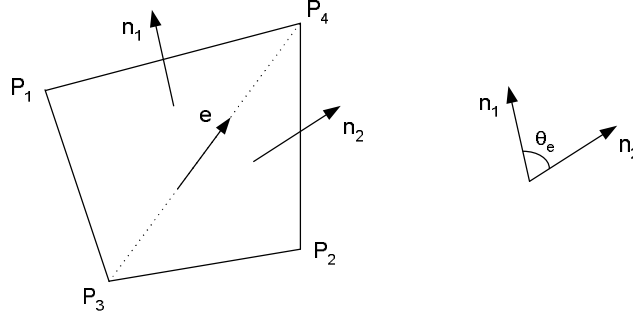
$$\frac{\partial C_A}{\partial P_{i(j,k)}} = a(P_{j(k,i)} - P_{k(i,j)}) \times \vec{A}$$

$$\frac{\partial^2 C_A}{\partial P_{i(j,k)}^2} = -a \left[(P_{j(k,i)} - P_{k(i,j)}) \right]^2$$

$$\frac{\partial^2 C_A}{\partial P_{i(j,k)} \partial P_{j(k,i)}} = -a \left[(P_{j(k,i)} - P_{k(i,j)}) \right] \left[(P_{k(i,j)} - P_{i(j,k)}) \right] - \left[\vec{A} \right]$$

$$\frac{\partial^2 C_A}{\partial P_{i(k,j)} \partial P_{k(j,i)}} = -a \left[(P_{j(i,k)} - P_{k(j,i)}) \right] \left[(P_{i(j,k)} - P_{j(k,i)}) \right] + \left[\vec{A} \right]$$

To keep curvature constant



Here all the vectors are normalized

$$C_B = \sqrt{\frac{\|\vec{e}\|}{h_e}} (\theta_e - \bar{\theta}_e)$$

$$\frac{\partial C_B}{\partial P} = \sqrt{\frac{\|\vec{e}\|}{h_e}} \cdot \frac{\partial \theta_e}{\partial P}$$

$$\frac{\partial \theta_e}{\partial P} = \frac{(\cos^2 \theta_e + \sin^2 \theta_e) \partial \theta_e}{\partial P} = \frac{\cos \theta_e \partial \sin \theta_e - \sin \theta_e \partial \cos \theta_e}{\partial P}$$

So, things are reduced to find $\frac{\partial \sin \theta_e}{\partial P}$ and $\frac{\partial \cos \theta_e}{\partial P}$

$$\cos \theta_e = n_1 \cdot n_2$$

$$\sin \theta_e = (n_1 \times n_2) \cdot e$$

$$\frac{\partial \theta_e}{\partial P} = \frac{\cos \theta_e \partial \sin \theta_e - \sin \theta_e \partial \cos \theta_e}{\partial P}$$

$$\frac{\partial \sin \theta_e}{\partial P} = \frac{\partial ((n_1 \times n_2) \cdot e)}{\partial P} = \left(\frac{\partial (n_1 \times n_2)}{\partial P} \right)^T e + \left(\frac{\partial e}{\partial P} \right)^T (n_1 \times n_2)$$

$$\frac{\partial \cos \theta_e}{\partial P} = \frac{\partial (n_1 \cdot n_2)}{\partial P} = \left(\frac{\partial n_1}{\partial P} \right)^T n_2 + \left(\frac{\partial n_2}{\partial P} \right)^T n_1$$

$$\frac{\partial (n_1 \times n_2)}{\partial P} = \left[n_1 \right] \frac{\partial n_2}{\partial P} - \left[n_2 \right] \frac{\partial n_1}{\partial P}$$

$$\frac{\partial^2 \theta_e}{\partial P_i \partial P_j} = \partial \left(\frac{\cos \theta_e \partial \sin \theta_e - \sin \theta_e \partial \cos \theta_e}{\partial P_i} \right) / \partial P_j$$

$$= \frac{\partial \sin \theta_e}{\partial P_i} \left(\frac{\partial \cos \theta_e}{\partial P_j} \right)^T + \cos \theta_e \frac{\partial^2 \sin \theta_e}{\partial P_i \partial P_j} - \frac{\partial \cos \theta_e}{\partial P_i} \left(\frac{\partial \sin \theta_e}{\partial P_j} \right)^T - \sin \theta_e \frac{\partial^2 \cos \theta_e}{\partial P_i \partial P_j}$$

$$\begin{aligned}
\frac{\partial^2 \sin \theta_e}{\partial P_i \partial P_j} &= \frac{\partial \left(\frac{\partial(n_1 \times n_2)}{\partial P_i} e + \frac{\partial e}{\partial P_i} (n_1 \times n_2) \right)}{\partial P_i \partial P_j} \\
&= \frac{\partial^2(n_1 \times n_2)}{\partial P_i \partial P_j} e + \frac{\partial(n_1 \times n_2)}{\partial P_i} \frac{\partial e}{\partial P_j} + \frac{\partial^2 e}{\partial P_i \partial P_j} (n_1 \times n_2) + \frac{\partial e}{\partial P_i} \frac{\partial(n_1 \times n_2)}{\partial P_j} \\
\frac{\partial^2 \cos \theta_e}{\partial P_i \partial P_j} &= \frac{\partial \left(\frac{\partial n_1}{\partial P_i} n_2 + \frac{\partial n_2}{\partial P_i} n_1 \right)}{\partial P_i \partial P_j} = \frac{\partial^2 n_1}{\partial P_i \partial P_j} n_2 + \frac{\partial n_1}{\partial P_i} \frac{\partial n_2}{\partial P_j} + \frac{\partial^2 n_2}{\partial P_i \partial P_j} n_1 + \frac{\partial n_2}{\partial P_i} \frac{\partial n_1}{\partial P_j} \\
\frac{\partial^2(n_1 \times n_2)}{\partial P_i \partial P_j} &= \frac{\partial \left([n_1] \frac{\partial n_2}{\partial P_i} - [n_2] \frac{\partial n_1}{\partial P_i} \right)}{\partial P_j} = \frac{\partial [n_1]}{\partial P_j} \frac{\partial n_2}{\partial P_i} + [n_1] \frac{\partial^2 n_2}{\partial P_i \partial P_j} - \frac{\partial [n_2]}{\partial P_j} \frac{\partial n_1}{\partial P_i} - [n_2] \frac{\partial^2 n_1}{\partial P_i \partial P_j}
\end{aligned}$$

In the above derivatives, $i, j = 1, 2, 3, 4$

Let's only look at the first triangle $\Delta P_1 P_3 P_4$, and as for the second triangle $\Delta P_2 P_4 P_3$, everything is similar.

For the first derivatives of face normal with respect to the vertices, we have

$$\left\{ \begin{array}{l} \frac{\partial n_1}{\partial P_1} = -\frac{[(P_3 - P_4)]}{2 \|A_{\Delta P_1 P_3 P_4}\|} \\ \frac{\partial n_1}{\partial P_2} = 0 \\ \frac{\partial n_1}{\partial P_3} = -\frac{[(P_4 - P_1)]}{2 \|A_{\Delta P_1 P_3 P_4}\|} \\ \frac{\partial n_1}{\partial P_4} = -\frac{[(P_1 - P_3)]}{2 \|A_{\Delta P_1 P_3 P_4}\|} \end{array} \right\}, \text{ and it follows that } \left\{ \begin{array}{l} \frac{\partial [n_1]}{\partial P_1} = \left[\frac{\partial n_1}{\partial P_1} \right] = -\frac{[(P_3 - P_4)]}{2 \|A_{\Delta P_1 P_3 P_4}\|} \\ \frac{\partial [n_1]}{\partial P_2} = \left[\frac{\partial n_1}{\partial P_2} \right] = 0 \\ \frac{\partial [n_1]}{\partial P_3} = \left[\frac{\partial n_1}{\partial P_3} \right] = -\frac{[(P_4 - P_1)]}{2 \|A_{\Delta P_1 P_3 P_4}\|} \\ \frac{\partial [n_1]}{\partial P_4} = \left[\frac{\partial n_1}{\partial P_4} \right] = -\frac{[(P_1 - P_3)]}{2 \|A_{\Delta P_1 P_3 P_4}\|} \end{array} \right.$$

Now the second derivatives

$$\left\{ \begin{array}{l} \frac{\partial^2 n_1}{\partial P_1^2} = \frac{\partial^2 n_1}{\partial P_1 \partial P_2} = \frac{\partial^2 n_1}{\partial P_2 \partial P_{1(2,3,4)}} = \frac{\partial^2 n_1}{\partial P_3 \partial P_2} = \frac{\partial^2 n_1}{\partial P_3^2} = \frac{\partial^2 n_1}{\partial P_4 \partial P_2} = \frac{\partial^2 n_1}{\partial P_4^2} = 0 \\ \frac{\partial^2 n_1}{\partial P_1 \partial P_3} = \frac{\partial^2 n_1}{\partial P_3 \partial P_4} = \frac{\partial^2 n_1}{\partial P_4 \partial P_1} = -\frac{I}{2 \|A_{\Delta P_1 P_3 P_4}\|} \\ \frac{\partial^2 n_1}{\partial P_1 \partial P_4} = \frac{\partial^2 n_1}{\partial P_3 \partial P_1} = \frac{\partial^2 n_1}{\partial P_4 \partial P_3} = \frac{I}{2 \|A_{\Delta P_1 P_3 P_4}\|} \end{array} \right.$$

Things are easy for the derivatives on the edge:

$$\left\{ \begin{array}{l} \frac{\partial e}{\partial P_1} = \frac{\partial e}{\partial P_2} = 0 \\ \frac{\partial e}{\partial P_3} = -\frac{I}{2 \|P_4 - P_3\|} \\ \frac{\partial e}{\partial P_4} = \frac{I}{2 \|P_4 - P_3\|} \end{array} \right.$$

And for all $i, j = 1, 2, 3, 4$, $\frac{\partial^2 e}{\partial P_i \partial P_j} = 0$

References

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